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GUIDANCE AND CONTROL SCHEMES FOR MANNED LUNAR LANDING

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By

C. N. Shen, Professor of Mechanical Engineering

Rensselaer Polytechnic Institute, Troy, N. Y.

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(1) Equations of Motion

The equations of motion(1) of a space vehicle in Fig 1 are written with the square of the specific angular momentum as the independent variable(2). These equations describe the dynamical behavior of the rocket-powered vehicle in an inverse square force field, i.e.,

$$\frac{d\theta}{dk} = \frac{u^3}{2a_\theta} , \qquad (1)$$

and

$$\frac{d^2u}{dk^2} + \left[\frac{1}{2k} + \frac{\frac{d}{dk}\left(\frac{2a_\theta}{u^3}\right)}{\left(\frac{2a_\theta}{u^3}\right)}\right] \frac{du}{dk} + \left(\frac{u^3}{2a}\right)^2 \left[u - \frac{g_o}{ku_o^2} + \frac{a_r}{ku^2}\right] = 0 \qquad (2)$$

where $u = 1/r$ = inverse of the radial distance measured from the center of the moon (or planet) to the space vehicle,

θ = angular displacement of the vector r with respect to the local vertical of the landing point,

= square of the specific angular momentum = $(r^2\dot\theta)^2$,

= radial specific force,

= transverse specific force,

= gravitational acceleration at a reference altitude

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above the moon (or planet),

$u_o = 1/r_o$ = inverse of the radius at a reference altitude above the moon (or planet).

(2) Variation Equations

If the specific forces (a_θ and a_r) are perturbed from the programmed values, the outputs of the vehicle, u and θ, will deviate from the programmed trajectory. In order to confine this deviation within a specified permissible range, proper guidance and control must be implemented. The perturbed forms of equations (1) and (2) are obtained by first substituting $u + \Delta u$, $\theta + \Delta\theta$, $a_\theta + \Delta a_\theta$, and $a_r + \Delta a_r$ for u, θ, a_θ, and a_r, respectively. The difference between the resulting equations and the original equations is determined and higher order terms involving Δu, $\Delta\theta$, Δa_θ and Δa_r are neglected. This procedure gives the following variational equations

$$\frac{d\Delta\theta}{dk} = 3\,\frac{u^3}{2a_\theta}\,\frac{\Delta u}{u} - \frac{u^3}{2a_\theta}\,\frac{\Delta a_\theta}{a_\theta}\,, \tag{3}$$

$$\frac{d^2\Delta u}{dk^2} + A(k)\,\frac{d\Delta u}{dk} + B(k)\Delta u = C(k)\Delta a_\theta + D(k)\Delta a_r + E(k)\,\frac{d\Delta a_\theta}{dk}\,, \tag{4}$$

where

$$A(k) = \frac{1}{2k} + \frac{\frac{d}{dk}\left(\frac{a_\theta}{u^3}\right)}{\left(\frac{a_\theta}{u^3}\right)} - \frac{3}{u}\,\frac{du}{dk}\,,$$

$$B(k) = 3\left(\frac{du}{dk}\right)^2 \frac{1}{u^2} + \frac{3u^6}{2a_\theta^{\ 2}}\left(1 - \frac{g_o}{ku_o^{\ 2}u} + \frac{a_r}{ku^3}\right) + \frac{u^6}{4a_\theta^{\ 2}} - \frac{u^3 a_r}{2a_\theta^{\ 2}k} ,$$

$$C(k) = \frac{1}{a_\theta}\left[\frac{3}{u}\left(\frac{du}{dk}\right)^2 + \frac{du}{dk}\frac{\frac{d}{dk}\left(\frac{a_\theta}{u^3}\right)}{\left(\frac{a_\theta}{u^3}\right)} + \frac{u^6}{2a_\theta^{\ 2}}\left(u - \frac{g_o}{ku_o^{\ 2}} + \frac{a_r}{ku^2}\right)\right],$$

$$D(k) = -\frac{u^4}{4ka_\theta^{\ 2}} ,$$

$$E(k) = -\frac{1}{a_\theta}\frac{du}{dk} ,$$

Δu is the perturbed quantity in u,

$\Delta\theta$ is the perturbed quantity in θ,

Δa_θ is the perturbed quantity in a_θ,

and Δa_r is the perturbed quantity in a_r.

These variational equations with k as an independent variable are ordinary linear differential equations with variable coefficients. The deviation $\Delta\theta$ and Δu at the landing point can be determined or computed from these equations.

(3) Computer Scheme

In general, the criteria for choosing a particular computer scheme are:

i The quantities a_θ and a_r must be finite during the entire descent trajectory.

ii The sources of information of the variables u and k are important. Subscripts m or k may be added to indicate that these variables are computed from measurements or other variables.

iii The nonlinear feedback should stabilize the system and force the vehicle into the reference trajectory.

In Fig. 2 and 3 the quantities u_m and k_m are the measured inverse radius and specific momentum of the vehicle, respectively. The computed inverse radius u_k is determined by a particular program in terms of k_m.

(4) The Angular Error $\Delta\theta$

Assuming no propulsion errors ($\delta a_\theta \to 0$, $\delta a_r \to 0$) and measurement (or guidance) errors ($\delta u \to 0$, $\delta k \to 0$), then

$$a_\theta = a_{\theta c} \text{ and } u_m = u, \tag{5a}$$

or

$$\Delta a_\theta = \Delta a_{\theta c} \text{ and } \Delta u_m = \Delta u. \tag{5b}$$

The variational equations for a_θ and $\frac{d\theta}{dk}$ can be written as

$$\Delta a_\theta = \Delta a_{\theta c} = \frac{\partial a_{\theta c}}{\partial u_m} \Delta u_m = \frac{\partial a_{\theta c}}{\partial u_m} \Delta u, \tag{6}$$

and

$$\frac{d}{dk} (\Delta\theta) = \frac{u^2 \Delta u}{2a_\theta} (3 - \frac{u}{a_\theta} \frac{\partial a_{\theta c}}{\partial u_m}). \tag{7}$$

If nonlinear guidance is employed in determining the trajectories, the computer is supposed to calculate the reference specific force according to a particular program, consistent with the criteria given previously in this section, hence

$$a_{\theta c} = \frac{{u_m}^3}{2\beta} , \tag{8a}$$

where β is a constant.

If $\delta a_\theta = \delta u = 0$, then $a_\theta = \frac{u^3}{2\beta}$. (8b)

By taking the partial derivative of equation (8a) one obtains

$$\frac{\partial a_{\theta c}}{\partial u_m} = \frac{3u_m^2}{2\beta} . \tag{9}$$

If equations (8a) and (9) are substituted into equation (7) we have

$$\frac{d}{dk} (\Delta\theta) = 0. \tag{10a}$$

If the initial perturbed quantity is $\Delta\theta_b$, then for the entire landing operation

$$\Delta\theta = \Delta\theta_b = \text{constant}. \tag{10b}$$

(5) The Radial Error Δu

The choice of the tangential specific force $a_{\theta c}$ in equation (8a) is very important not only for the simplification of the computation of the angular error but also for the determination of the radial error. Equations (5), (6) and (9) show that

$$\Delta a_\theta = \frac{3u^2}{2\beta} \Delta u, \tag{11}$$

from which one obtains

$$\frac{d}{dk} (\Delta a_\theta) = \frac{3u}{2\beta} [\frac{du}{dk}2(\Delta u) + u \frac{d}{dk} (\Delta u)]. \tag{12}$$

The differential equation for the radial error Δu can be greatly simplified by substituting equations (8b), (11) and (12) into equation (4). The result is,

$$\frac{d^2}{dk^2} (\Delta u) + \frac{1}{2k} \frac{d}{dk} (\Delta u) + \beta^2[1 - \frac{2a_r}{ku^3}](\Delta u) = - \frac{\beta^2}{ku^2} \Delta a_r. \tag{13}$$

The computer program for the radial specific force (a_{rc}) has the functional form

$$a_{rc} = \phi(u_m, u_k, k_m), \tag{14}$$

where the subscripts m and k are assigned to u to designate the signal sources. The feedback, which is shown in Figs. 2 and 3, must be properly designed to ensure system stability. The variational form of equation (14) is

$$\Delta a_{rc} = \frac{\partial a_{rc}}{\partial u_m} \Delta u_m + \frac{\partial a_{rc}}{\partial u_k} \frac{du_k}{dk_m} \Delta k_m + \frac{\partial a_{rc}}{\partial k_m} \Delta k_m. \tag{15}$$

As indicated in equation (15) the perturbed quantity, Δa_{rc} results from the perturbations Δu_m and Δk_m. If the k measurement error is zero, then

$$k = k_m. \tag{16}$$

Since k is an independent variable in the analysis, it follows that

$$\Delta k = \Delta k_m = 0. \tag{17}$$

Hence equation (15) can be reduced to the following:

$$\Delta a_{rc} = \frac{\partial a_{rc}}{\partial u_m} \Delta u_m. \tag{18}$$

Equation (13) shows that the perturbed quantity Δu is influenced by the forcing function which is generated by the perturbed radial specific force Δa_r. If it is further assumed that there is no propulsion error in a_r, then

$$a_{rc} = a_r, \tag{19a}$$

which implies

$$\Delta a_{rc} = \Delta a_r. \tag{19b}$$

With equations (5b), (18), (19a) and (19b), equation (13) reduces to the homogeneous differential equation,

$$\frac{d^2\Delta u}{dk^2} + \frac{1}{2k}\frac{d\Delta u}{dk} + \beta^2\left[1 - \frac{2a_{rc}}{k_m u_m^3} + \frac{1}{k_m u_m^2}\frac{\partial a_{rc}}{\partial u_m}\right]\Delta u = 0. \qquad (20)$$

The radial specific force a_r is chosen to be

$$a_r = a_{rc} = \frac{g_o u_m^2}{u_o^2} - k_m u_m^3 + \lambda^2 k_b u_m^2 (u_k - u_o), \qquad (21)$$

in accordance with the criteria given previously in this section. Differentiating equation (21) with respect to u_m, one obtains the value of Δa_r in equation (18). Substituting this value and equation (21) into equation (20) yields

$$\frac{d^2\Delta u}{dk^2} + \frac{1}{2k}\frac{d\Delta u}{dk} = 0. \qquad (22)$$

The solution of equation (22) is

$$\Delta u = c_1 k^{\frac{1}{2}} + c_2, \qquad (23)$$

where c_1 and c_2 are constants of integration.

(6) Terminal Altitude Error Δr

By employing the proposed feedback program for a_{rc} and $a_{\theta c}$, the effects due to the different initial errors on the terminal altitude error can be determined. The first case begins with the assumption that the vehicle is in a circular orbit with an initial radial error of Δu_b before it starts to descend. The errors in the initial conditions are expressed as

$$\Delta u \Big|_{k=k_b} = \Delta u_b \tag{24}$$

$$\frac{d\Delta u}{dk} \Big|_{k=k_b} = 0 \tag{25}$$

With equations (24) and (25), equation (23) yields

$$c_1 = 0 \qquad \text{and} \qquad c_2 = \Delta u_b$$

Thus the solution becomes

$$\Delta u = \Delta u_b \tag{26}$$

With the aid of the definition $u = \frac{1}{r}$, it leads to

$$\Delta u = -\frac{1}{r^2}\Delta r \tag{27}$$

After substituting equation (27) into equation (26), the altitude error is

$$\Delta r = \left(\frac{r}{r_b}\right)^2 \Delta r_b \tag{28}$$

where Δr_b is the initial altitude error.

The circular parking orbit of 22.04-mile altitude corresponds to r_b = 1102.04 miles, while the radius of the lunar surface is r_o = 1080 miles. If Δr_b = 0.1 mile, then the terminal altitude error $\Delta r \Big|_{k=0}$ = 0.096 mile.

For trajectories other than circular orbit one may start with a correct altitude (i.e., $\Delta u = 0$) but with an angular deviation other than the programmed slope. Then the initial errors resulting from the deviations are

$$\Delta u \Big|_{k=k_b} = 0 \tag{29}$$

$$\frac{d\Delta u}{dk} \Big|_{k=k_b} = W_b \tag{30}$$

Using equations (29) and (30), the constants of integration in equation (23) can be determined. Thus, the solution for equation (23) is

$$\Delta u = 2W_b(k\ k_b)^{\frac{1}{2}} - 2\ W_b k_b \tag{31}$$

which is valid for the entire landing operation. The particular value of k is zero at landing, thus

$$\Delta u \Big|_{k=0} = -2\ W_b k_b \tag{32}$$

To interpret the quantity W_b in equation (30) it is desirable to evaluate the derivative of Δu with respect to k in equation (27).

$$\frac{d\Delta u}{dk} = \frac{2}{r^3}\frac{dr}{dk}\Delta r - \frac{1}{r^2}\frac{d\Delta r}{d\theta}\frac{d\theta}{dk} \tag{33}$$

Since $\Delta r \Big|_{k=k_b} = \Delta u \Big|_{k=k_b} = 0$ by equation (29), therefore,

$$W_b = -\frac{1}{{r_b}^2}\left[\frac{d\Delta r}{d\theta}\frac{d\theta}{dk}\right]_{k=k_b} \tag{34}$$

From equations (1) and (8.), we have

$$\frac{d\theta}{dk} = \beta \tag{35}$$

The value of β can be determined from the trajectory for $n=0$, (see Appendix) as

$$\beta = \frac{\theta_b}{k_b} \tag{36}$$

Thus

$$W_b = -\frac{1}{r_b^{2}}\frac{\theta_b}{k_b}\frac{d\Delta r}{d\theta}\bigg|_{k=k_b} = -\frac{1}{r_b^{2}}\frac{\theta_b r_b}{k_b}\Delta\left(\frac{dr}{ds}\right)\bigg|_{k=k_b} \tag{37}$$

The terminal altitude error Δr is determined from equations (27), (32) and (37) as

$$\Delta r\bigg|_{k=0} = -2\left(\frac{r_o}{r_b}\right)^{2}\theta_b r_b \Delta\left(\frac{dr}{ds}\right)\bigg|_{k=k_b} \tag{38}$$

where $ds = r_b d\theta$ and $\Delta\left(\frac{dr}{ds}\right)\bigg|_{k=k_b}$ is an angular deviation from the programmed path.

Fig. 4 shows the relationship between the terminal altitude error and the landing angle θ_b for the different angular deviations at initial point.

(7) Error of the Terminal Transverse Velocity

Based on the definition of k, the error of the transverse velocity can be determined. Since

$$k = \left(r^2 \frac{d\theta}{dt}\right)^2 = (r v_\theta)^2 \tag{39}$$

the transverse velocity of the vehicle $(v_\theta = r \frac{d\theta}{dt})$ can also be written as

$$v_\theta = - k^{\frac{1}{2}} u \tag{40}$$

With k taken as the reference variable in the feedback control, the perturbed v_θ and the perturbed u have the following relation

$$\Delta v_\theta = - k^{\frac{1}{2}} \Delta u \tag{41}$$

and

$$\Delta v_\theta \Big|_{k=0} = 0 \tag{42}$$

(8) Error of the Terminal Radial Velocity

It is of great importance to investigate the error of the terminal radial velocity because of the stringent condition imposed on the terminal touch-down velocity in achieving soft landing. It is defined that $v_r = \frac{dr}{dt}$

Hence,

$$v_r = - \frac{du}{u^2 dt} = - \frac{1}{u^2} \frac{d\theta}{dt} \frac{du}{d\theta}$$

$$= - k^{\frac{1}{2}} \frac{du}{d\theta} \tag{43}$$

The effect on Δv_r due to the values Δu and $\Delta\theta$ is determined as

$$\Delta v_r = -k^{\frac{1}{2}}\left[\frac{d(u+\Delta u)}{dk}\left(\frac{d\theta+\Delta\theta}{dk}\right)^{-1} - \frac{du}{dk}\left(\frac{d\theta}{dk}\right)^{-1}\right] \tag{44}$$

Thus

$$\Delta v_r = -k^{\frac{1}{2}}\frac{d\Delta u}{dk}\left(\frac{d\theta}{dk}\right)^{-1} \tag{45}$$

where $\frac{d\Delta\theta}{dk} = 0$ as shown in equation (10a). The trajectory of the vehicle has a simple analytical expression when the value n equals zero, which implies (see Appendix A)

$$\frac{d\theta}{dk} = \beta = \frac{\theta_b}{k_b}$$

Thus

$$\Delta v_r = -\frac{k_b}{\theta_b}\frac{d\Delta u}{dk}k^{\frac{1}{2}} \tag{46}$$

To evaluate the error of the radial velocity Δv_r, it is required to calculate the value $\frac{d\Delta u}{dk}$, which is obtained from equation (23)

$$\frac{d\Delta u}{dk} = \frac{1}{2}c_1 k^{-\frac{1}{2}} \tag{47}$$

Equation (46) is reduced further as

$$\Delta v_r = -\frac{1}{2}\frac{k_b}{\theta_b}c_1 \tag{48}$$

which shows that the value of Δv_r remains constant throughout the entire landing operation.

It is to be noted that from equation (47) $c_1 = 0$ if the initial conditions are

$$\left.\frac{d\Delta u}{dk}\right|_{k=k_b} = 0 \qquad \text{and} \qquad \left.\Delta u\right|_{k=k_b} = \Delta u_b$$

Therefore $\Delta v_r = 0$ for the above case. However, for the case where $\left.\frac{d\Delta u}{dk}\right|_{k=k_b} = W_b$ and $\left.\Delta u\right|_{k=k_b} = 0$, we have

$$c_1 = 2W_b k_b^{\frac{1}{2}} \tag{49}$$

which is obtained by comparing equations (23) and (31). Substituting the value of W_b from equation (37) into equation (49), one obtains

$$c_1 = -2\,\frac{\theta_b k_b^{-\frac{1}{2}}}{r_b^2}\, r_b\, \Delta\left(\frac{dr}{ds}\right)\bigg|_{k=k_b} \tag{50}$$

From equation (50) and equation (48), we have

$$\Delta v_r = \frac{k_b^{\frac{1}{2}}}{r_b}\,\Delta\left(\frac{dr}{ds}\right)\bigg|_{k=k_b} = v_{\theta b}\Delta\left(\frac{dr}{ds}\right)\bigg|_{k=k_b} \tag{51}$$

where $v_{\theta b}$ is the velocity of the vehicle in the circular orbit. Fig. 5 shows the linear relationship between the error of the radial velocity (Δv_r) and the angular deviation $[\Delta(\frac{dr}{ds})]$ of the initial trajectory with respect to the circular orbit. For example, an angular deviation of 3 minutes will give an error of radial velocity of 3.25 mile/hr.

27392

CONCLUSIONS

(a) The objective of this paper is to find feasible control schemes for descent trajectories that will give, with high accuracy, the final position and velocity for lunar landing. A stable control system can be provided for any initial error. With the aid of a feedback system obtained from measurements of the state of the vehicle very small deviations from the programmed course are expected so that a bull's eye landing can be achieved.

(b) The variational equations (3) and (4) are completely general in scope for any two-dimensional problem provided the vehicle is not coasting, i.e., $k \neq$ constant.

(c) By choosing the computer scheme and the non-linear guidance for specific forces a_θ and a_r, the angular and radial errors are constants and the velocity errors are zero if the system starts with the specific perturbed initial conditions.

(d) The solution for the reference trajectory is in algebraic closed form, which simplifies the computational aspect of the control, either by manual operation or automatic.

Author

FUTURE STUDY

(1) Other forms of perturbation.

A feedback systems for damping the initial angular and radial error will be investigated, for example, perturbed velocity feedback. As the dynamic process is a multiple variable system with coupling effects between the angular and radial components, the characteristics of the output must be clearly understood.

(2) The stability of the feedback system will be studied. Since the equations of the dynamic system are in terms of the independent variable k instead of time, the stability criterion must be redefined since k decreases monotonically with increasing time.

(3) If an earth bound signal is used, the computers for $a_{\theta c}$ and a_{rc} can make calculations on earth and send a command signal to the lunar bug. A delay$(\tau_1 + \tau_2 + \tau_3)$ of at least 3 seconds is shown in Fig. 3. The effect of this "pure delay" will be studied.

(4) Statistical error in measurements and guidance appear as δk and δu and in the propulsion system as δa_θ and δa_r in Fig. 3. The control of this nonlinear stoicastic process will be analyzed.

ACKNOWLEDGEMENT

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APPENDIX A

The solutions to Equations (1) and (2) in the text can be obtained provided that the specific forces are given. The transverse and radial specific forces must be chosen so that the soft landing requirements are satisfied. It is purposed that

$$\left(\frac{2a_\theta}{u^3}\right) = \frac{1}{\beta}\left(\frac{k}{k_b}\right)^n , \qquad 0 \leq n < 1, \tag{A1}$$

$$\left(\frac{a_r}{ku^2}\right) = \frac{g_0}{ku_0^2} - u + \lambda^2 \frac{k_b^{2q}}{k^{2q}}(u-u_0) , \qquad q \leq \frac{1}{2} , \tag{A2}$$

where k_b is the initial value of k.

The constants parameters λ and β are to be determined. The justification for the limits on the parameters on n and q is that, as k approaches zero, the value of a_θ and a_r must be finite. This results in $0 \leq n$ and $q \leq \frac{1}{2}$. Under the initial conditions ($\theta = \theta_b$ at $k = k_b$) and the final conditions ($\theta = 0$ at $k = 0$), the following expressions are obtained by solving equations (1) and (A1)

$$\left(\frac{\theta}{\theta_b}\right)^{\frac{1}{1-n}} = \left(\frac{k}{k_b}\right), \tag{A3}$$

and

$$\beta = \frac{(1-n)\theta_b}{k_b} \tag{A4}$$

The solution to equation (2) after substituting (A1) and (A2) into it with $q = \frac{1}{2}$ and $n = 0$ is

$$\frac{U}{U_b} = \frac{\sin 2\lambda\theta_b \left(\frac{\theta}{\theta_b}\right)^{\frac{1}{2}}}{\sin 2\lambda\theta_b} \tag{A5}$$

where $U = u_o - u$.

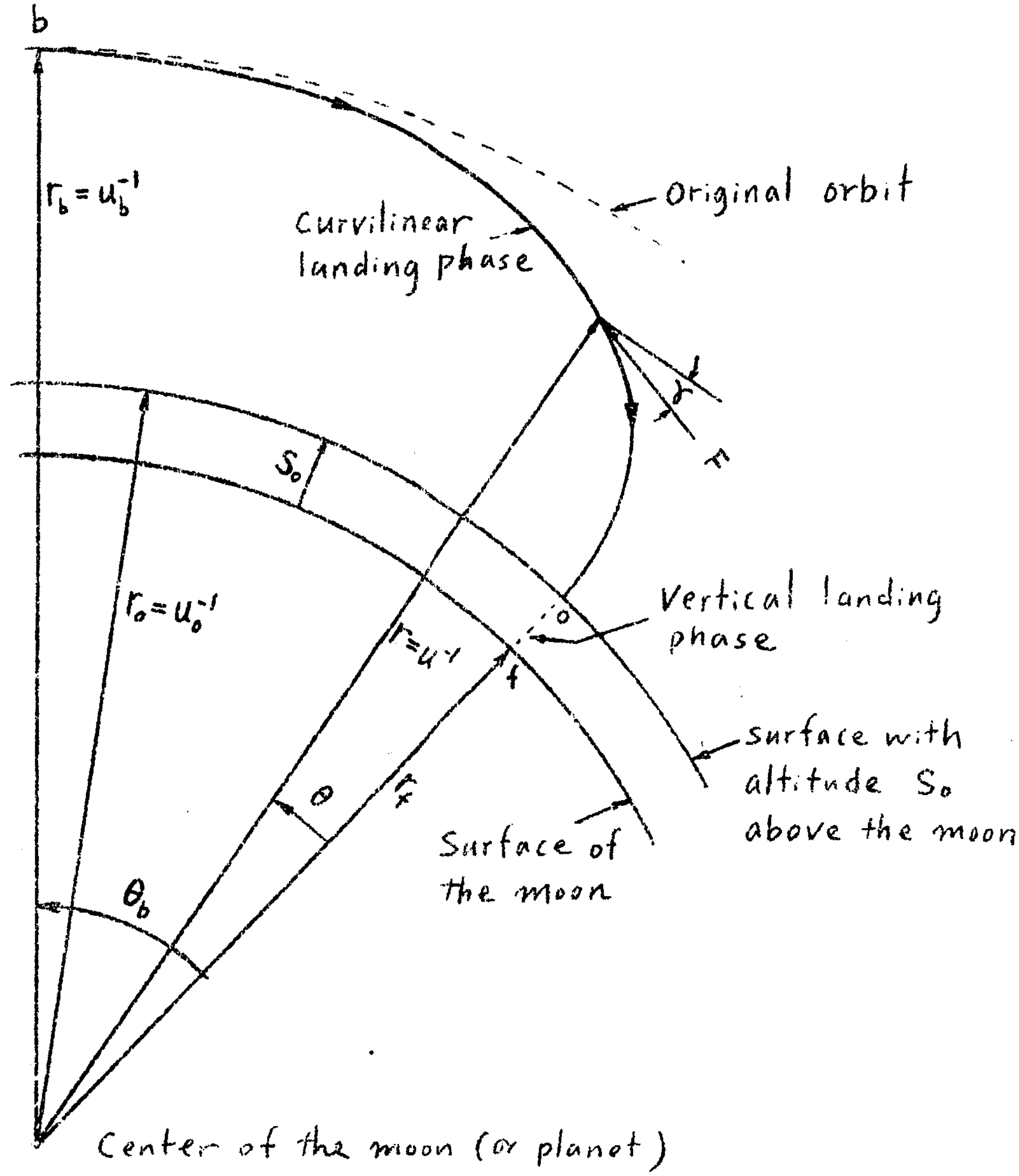

Fig 1 Descent Trajectory

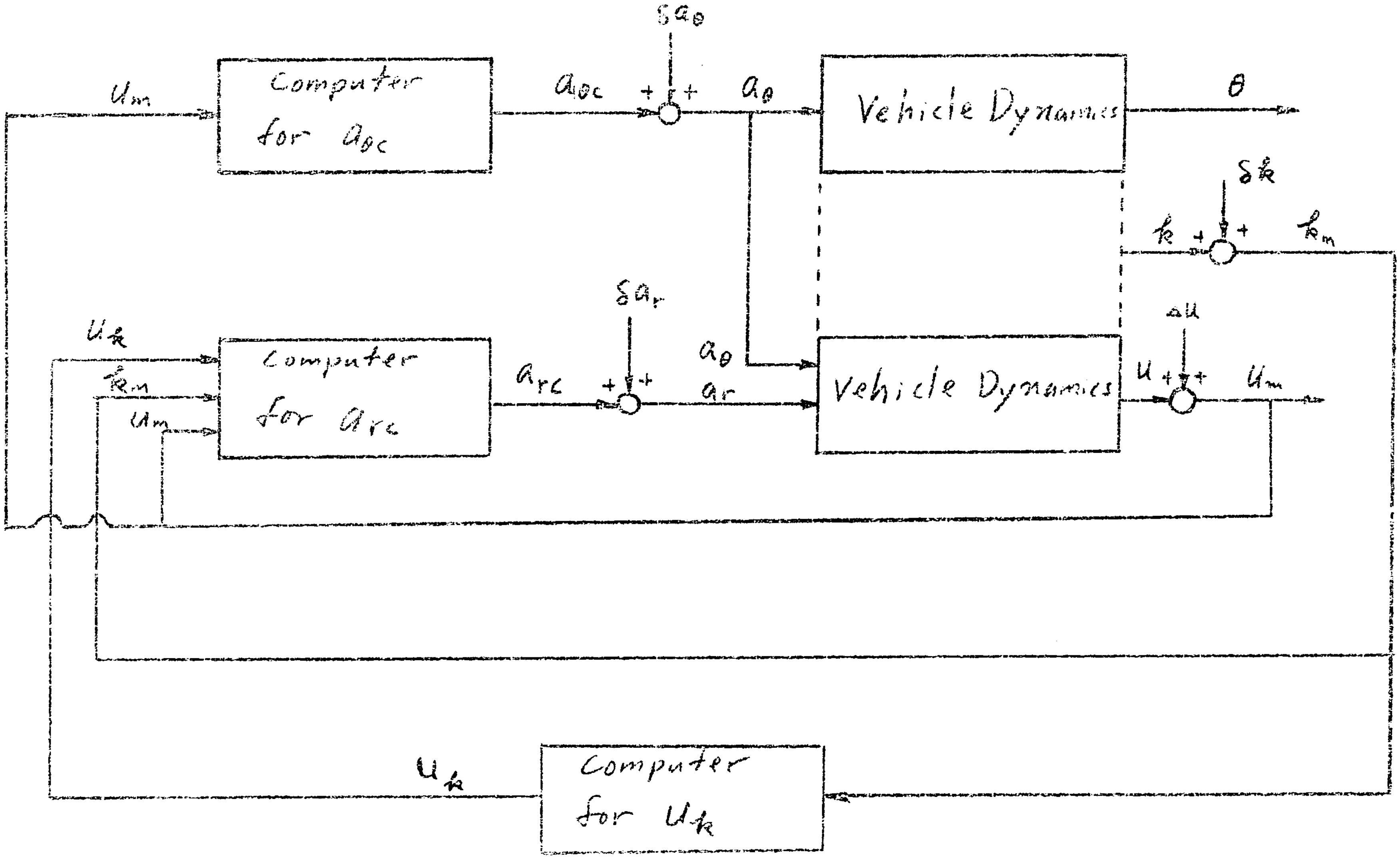

Fig 2 Computer Scheme

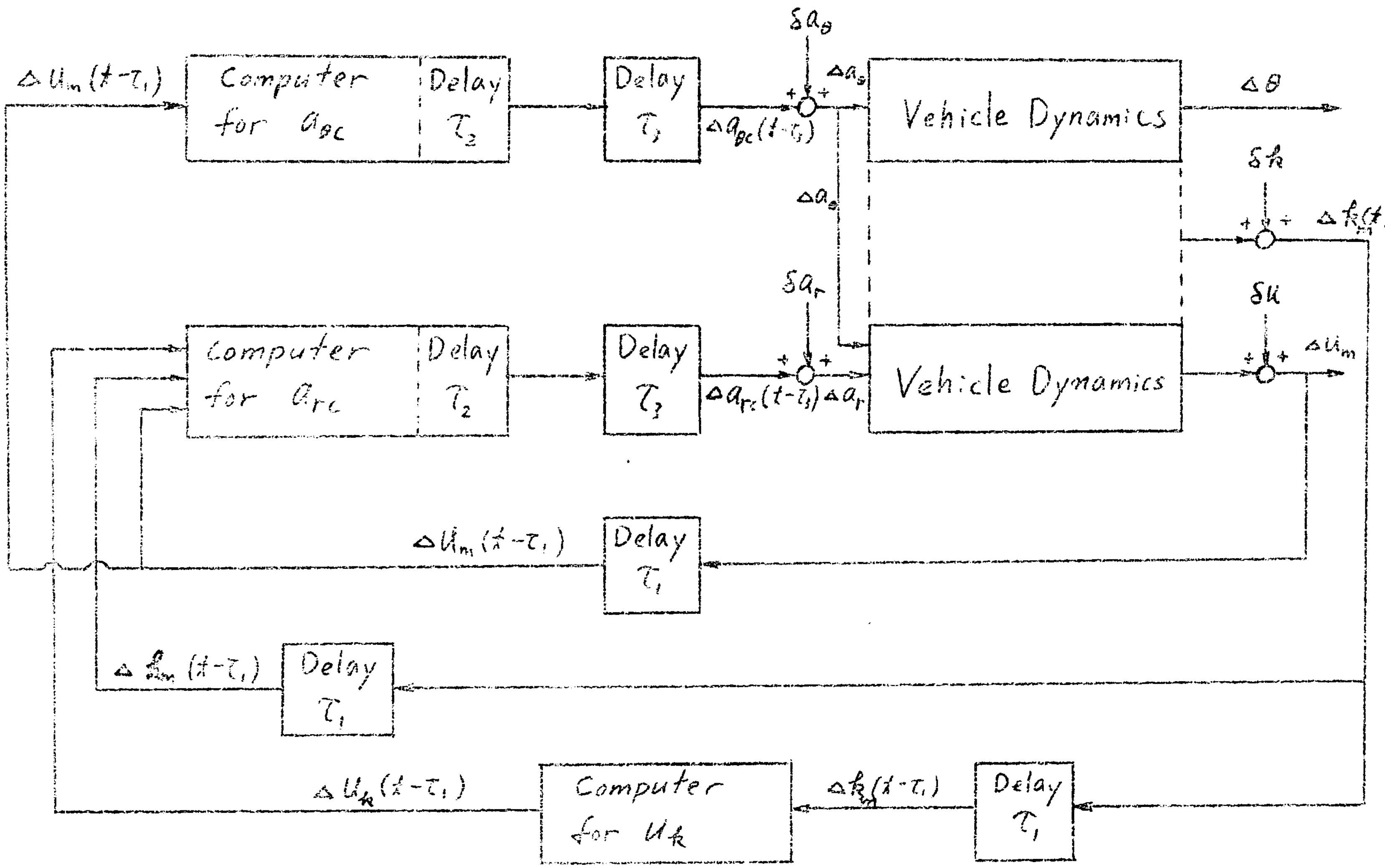

Fig 3 Computer Scheme

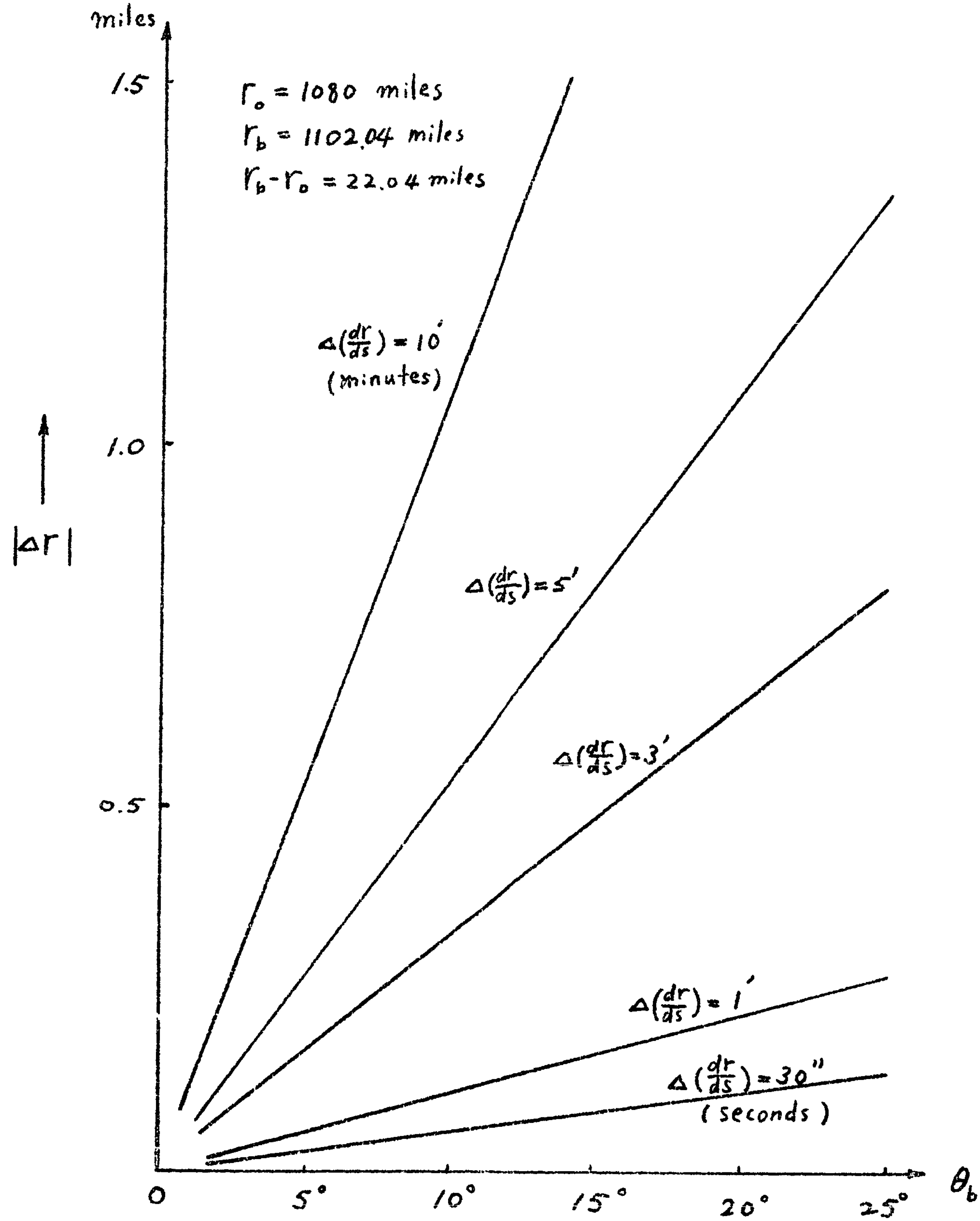

Fig 4 Terminal Altitude Error vs Landing Angle θ_b

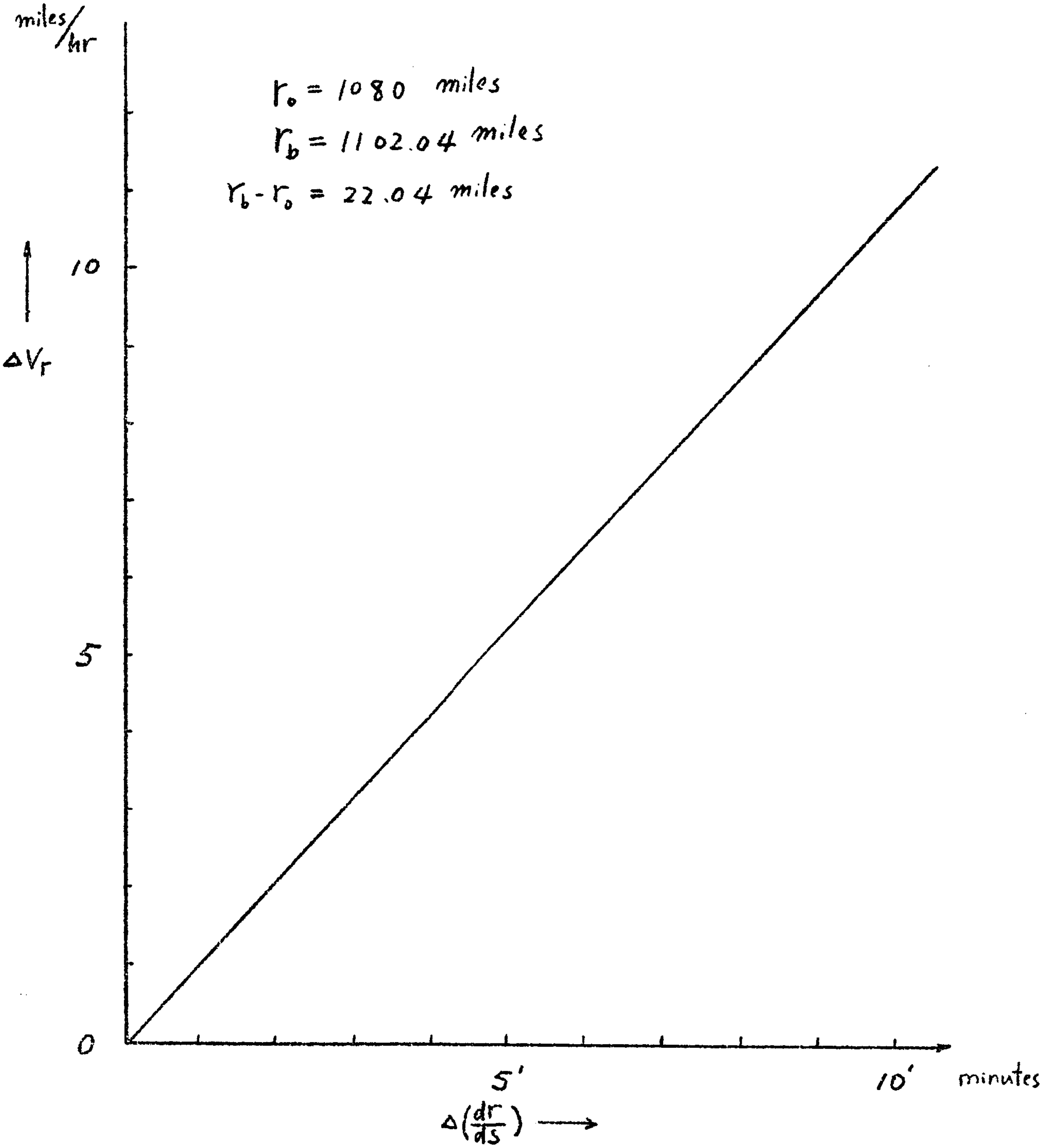

Fig 5. Error of the radial velocity vs Angular deviation $\Delta(\frac{dr}{ds})$